

Response of a DC RC circuit in series embedded with a rotating varco

Sparisoma Viridi*

Nuclear Physics and Biophysics Research Division
Institut Teknologi Bandung, Bandung 40132, Indonesia

Siti Nurul Khotimah†

Nuclear Physics and Biophysics Research Division
Institut Teknologi Bandung, Bandung 40132, Indonesia

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Abstract

Response in electric potential U from a DC RC circuit in series has been obtained by varying the capacitance of the capacitor C . Variation of the capacitance begins at $t_0 > 0$. Alternating response is observed when the periode of change of capacitance in the same order as $\tau = RC$, which is held constant for $t < t_0$ and varied in triangle form for $t > t_0$.

Keywords: DC circuit, RC circuit, rotating varco, response.

*dudung@fi.itb.ac.id

†nurul@fi.itb.ac.id

1 Introduction

In a simple direct current circuit (DC) in series consisted of a battery, a resistor, and a capacitor current response is normally exponentially decayed to zero with increasing of time [1]. Alternating current response can be obtained when the voltage source has a alternating current (AC) type [2]. An AC response in a DC resistor-capasitor (RC) circuit in series can be also observed when the capacitance of the capacitor is varied with time, as reported in this work.

2 Variabel condensator

A variable condensator or also known as varco is a capacitor that its capacitance can be varied. One type is varied by turning its rotor clockwise and counterclockwise [3]. Suppose that a varco consists of a pair of half circle plate with diameter D then its capacitance can be defined as

$$C = \frac{\epsilon_0 D^2}{8d} |\pi - \theta|, \quad (1)$$

with d is the distance between pair of the half circle plate and ϵ_0 is vacuum permittivity. The angle θ is periodic in $[0, 2\pi]$. If the varco rotor is rotating with uniform angular velocity ω then the angular position is

$$\theta = \omega t. \quad (2)$$

Substitution of Equation (1) into Equation (2) will give the capacitance of the varco as function of time

$$C = \frac{\epsilon_0 D^2}{8d} |\pi - \omega t|. \quad (3)$$

3 A DC RC circuit in series

A DC RC circuit in series consisted of battery ε , resistor R , and capacitor C has a relation

$$\varepsilon - IR - \frac{Q}{C} = 0, \quad (4)$$

with relation between current I and charge Q is

$$I = \frac{dQ}{dt}. \quad (5)$$

Derivation of Equation (4) using (5) will give the relation

$$\frac{dI}{dt} + \frac{I}{\tau} - \frac{Q}{C\tau} \frac{dC}{dt} = 0, \quad (6)$$

with

$$\tau = RC. \quad (7)$$

Substituting expression of Q/C from Equation (4) into Equation (6) will give

$$\frac{dI}{dt} = - \left(\frac{1}{\tau} + \frac{R}{\tau} \frac{dC}{dt} \right) I + \frac{\varepsilon}{\tau} \frac{dC}{dt}, \quad (8)$$

that will reduce to known result [1] for constant capacitance, $dC/dt = 0$. In this case expression of dC/dt can be found from Equation

$$\frac{dC}{dt} = \frac{\epsilon_0 D^2}{8d} \cdot \begin{cases} 0, & t < t_0 \\ -\omega, & t_0 + 2n\pi/\omega \leq t < t_0 + (2n+1)\pi/\omega \\ \omega, & t_0 + (2n+1)\pi/\omega \leq t < t_0 + 2(n+1)\pi/\omega \end{cases}, \quad (9)$$

with t_0 is start time of rotating varco. It is chosen that the capacitance

$$C = \frac{\epsilon_0 D^2}{8d} \cdot \begin{cases} \pi, & t < t_0 \\ \pi - \omega t, & t_0 + 2n\pi/\omega \leq t < t_0 + (2n+1)\pi/\omega \\ \omega t - \pi, & t_0 + (2n+1)\pi/\omega \leq t < t_0 + 2(n+1)\pi/\omega \end{cases} \quad (10)$$

Then it can also be obtained that

$$\tau = \frac{R\epsilon_0 D^2}{8d} \cdot \begin{cases} \pi, & t < t_0 \\ \pi - \omega t, & t_0 + 2n\pi/\omega \leq t < t_0 + (2n+1)\pi/\omega \\ \omega t - \pi, & t_0 + (2n+1)\pi/\omega \leq t < t_0 + 2(n+1)\pi/\omega \end{cases} \quad (11)$$

4 Numerical solution

Equation (8) with Equation(9)-(11) can be solved numerically, using *e.g.* Euler method [4], which is

$$I(t + \Delta t) = I(t) + \left(\frac{dI}{dt} \right) \Delta t. \quad (12)$$

in this case. It is chosen that

$$\Delta t = 10^{-3} \frac{2\pi}{\omega}. \quad (13)$$

5 Results and discussion

Following parameters are used in the calculation: $R = 1 \text{ k}\Omega$, $\varepsilon = 10 \text{ V}$, $t_0 = 2 \text{ }\mu\text{s}$, $D = 0.5 \text{ m}$, $d = 1 \text{ mm}$, and $\Delta t = 10 \text{ ns}$. Frequency, $f = \omega/2\pi$,

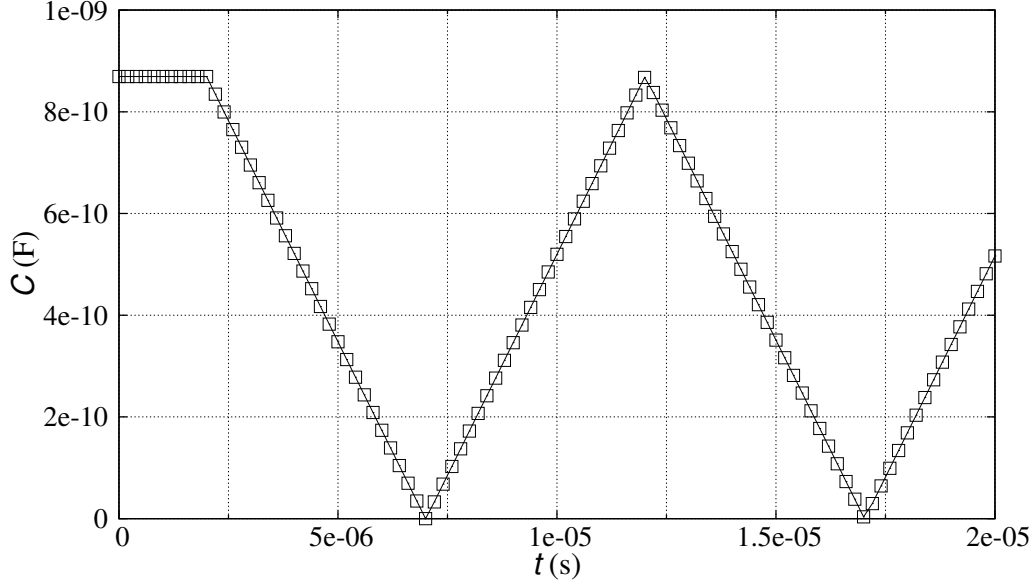


Figure 1: . A typical form of C for $f = 10^5$ Hz.

is varied: 10 Hz - 1 MHz. It is found that $C(t < t_0) = 0.869258$ nF and $\tau(t < t_0) = 0.869258$ ms.

Response in form of $U = IR$ is given in Figure 3, which illustrates that only periode of rotating varco that nearly the same order as τ could give the alternating response as $f = 10^5$ and 10^6 Hz.

In the calculation detail an exception must be implemented to avoid value of $\tau = 0$ since it is allowed as given by $C = 0$ as illustrated in Figure 1 and defined in Equation (11).

6 Conclusion

An alternating response can be obtained from a DC RC circuit in series by varying the value of capacitance of the capacitor which is assume to be a varco. The variation is taken place by rotating the rotor of the varco with a constant angular velocity. Only rotation periode that has the same order as

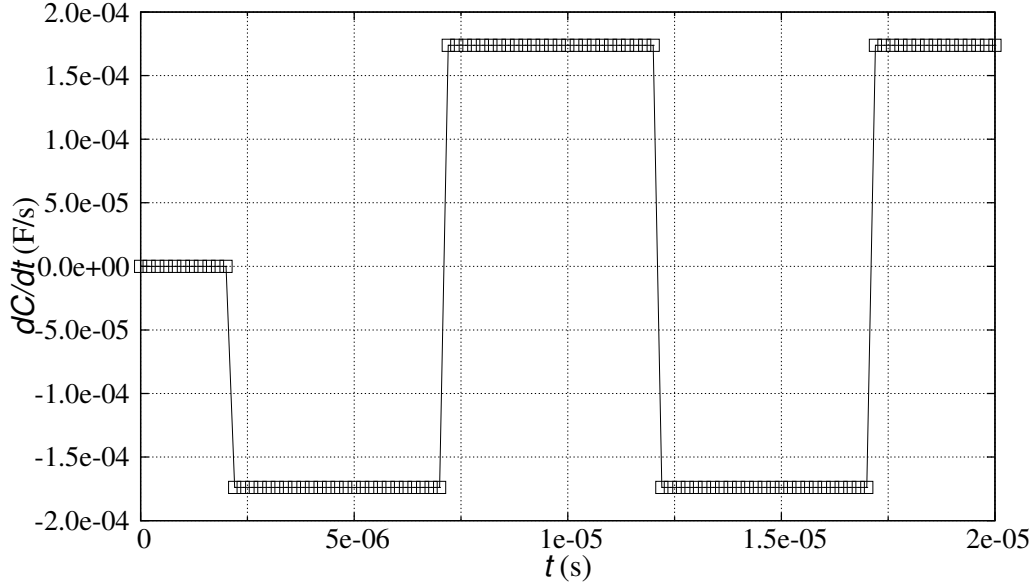


Figure 2: . A typical form of dC/dt for $f = 10^5$ Hz.

$\tau = RC$ will give an alternating response.

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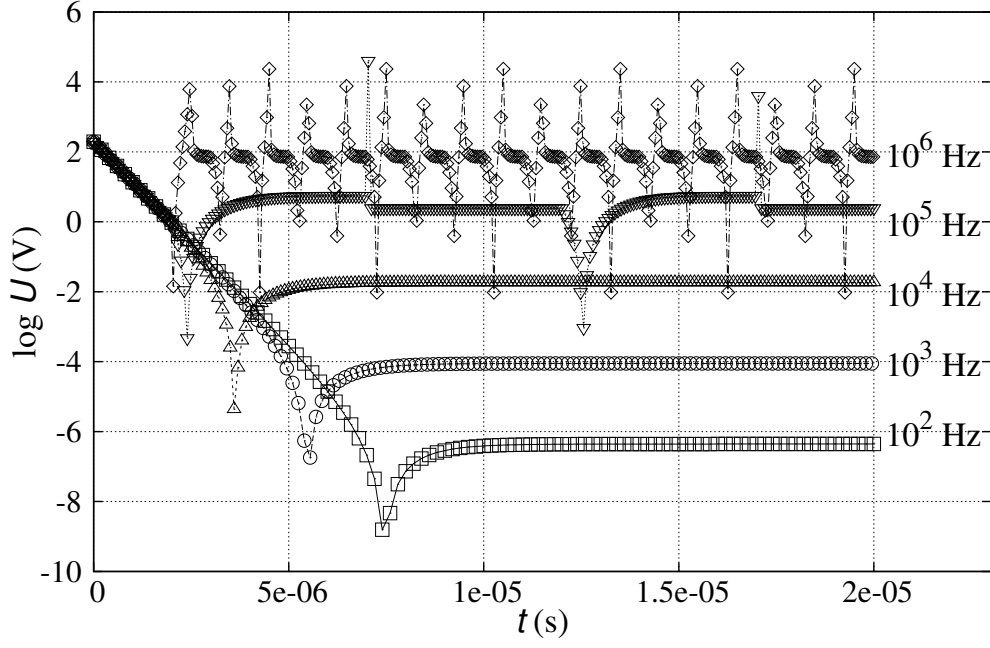


Figure 3: . Response of $\log U$ against time t for several frequency $f = 10^2$, 10^3 , 10^4 , 10^5 , and 10^6 Hz.

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